Available online at www.sciencedirect.com
ScienceDirect
Computers and Mathematics with Applications 53 (2007) 80–88
An International Journal
computers & mathematics with applications
www.elsevier.com/locate/camwa

Optimal tracking control for large-scale interconnected systems with time-delays

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Received 16 April 2006; received in revised form 29 June 2006; accepted 12 July 2006

Abstract

This paper considers an infinite-horizon optimal tracking control problem for a class of large-scale interconnected systems with state time-delays. By using the successive approximation approach, two iteration sequences of vector differential equations are constructed. Meanwhile the large-scale interconnected system is decomposed into finite decoupled subsystems. The existence and uniqueness of the optimal solution is proved, as well as the convergence of the solution sequence. By finite iterations of the solution sequence, a suboptimal tracking control law is obtained. A reduced-order reference input observer is designed to make the feedforward term of the optimal tracking control law physically realizable. A numerical example shows that the presented algorithm is effective and easy to implement.

Keywords: Tracking control; Optimal control; Time-delay; Large-scale systems; Successive approximation approach

1. Introduction

Time-delays are frequently encountered in various engineering systems, such as networked control systems [1], chemical processes and heat exchange systems [2]. Time-delays often cause deterioration in control system performance and may be a source of instability. During the past several years, delay-dependent stability of linear systems has attracted considerable attention [3–7]. The goal is to obtain the maximum allowed upper bound on the delay that guarantees the stability of a linear neutral system. As pointed out in [6], there are two kinds of time-delay systems, i.e. systems with small delay and systems with non-small delay. For the former, a model transformation technique is often used to transform a pointwise (discrete) delay system into a distributed delay system, and delay-dependent stability criteria can be obtained using the bounding technique for cross terms. However, the model transformation may introduce additional dynamics. In order to reduce the conservatism, Han [6,7] proposed some new methods to avoid using model transformation and bounding technique for cross terms.

The optimal control of time-delay systems has been widely investigated in recent decades. The quadratic optimal control of time-delay systems generally leads to solving a two-point boundary value (TPBV) problem with both time-delay and time-advance terms, which is very difficult to solve analytically with the exception of some simplest

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0898-1221/$ - see front matter
doi:10.1016/j.camwa.2006.07.016

cases. So it is necessary to find an approximate optimal control law, and many methods have been proposed recently. One of them is the power series approximation approach, which uses a power series expansion [8], or Adomian's decomposition [9] to approximate the solution of the Hamilton–Jacobi–Bellman (HJB) equation. A second method is the successive Galerkin approximation [10], which reduces the HJB equation to a sequence of linear partial differential equations. Another approach is called the state-dependent Riccati equation [11,12], which requires the solution of the Riccati equation at each given point of the system state. In order to simplify the computation, Aganovic and Gajic [13] proposed a method of successive approximation, where only solutions of a sequence of differential Lyapunov equations are required. However, since these algorithms are based on iterations of matrix differential equations, they usually take long computing time and large memory space, which becomes worse under circumstances where the dynamical systems have higher dimensions and more subsystems. Therefore, they are usually inadequate for time-critical applications, which motivates the present study.

The objective of this paper is to design an infinite-horizon optimal tracking controller for a class of large-scale interconnected systems with state time-delays and to find a new algorithm with low computational complexity. Based on the successive approximation approach, two iteration sequences of vector differential equations are first constructed. Meanwhile the large-scale interconnected system is decomposed into finite decoupled subsystems. The existence and uniqueness of the optimal solution is proved, as well as the convergence of the solution sequence. Furthermore, by finite iterations of the solution sequence, an approximate solution is obtained. Finally, a reduced-order reference input observer is designed to make the feedforward term of the optimal tracking control law physically realizable.

2. Problem statement

Consider a class of large-scale interconnected systems with state time-delays. It is assumed that the system consists of N interconnected subsystems and the ith subsystem is described by

$$\begin{aligned} &\dot{x}_i(t) = A_{ii}x_i(t) + A_i x(t) + B_i u_i(t) + D_i x_i(t-\tau_i), \quad t > 0 \\ &x_i(t) = \phi_i(t), \quad -\tau_i \le t \le 0 \\ &y_i(t) = C_i x_i(t), \quad i = 1, 2, \dots N, \end{aligned} \tag{1}$$

where $x_i \in R^{n_i}$ is the state vector, $u_i \in R^{r_i}$ is the control input and $y_i \in R^{m_i}$ is the measured output, respectively; $x = [\, x_1^{\mathrm{T}} \;\; x_2^{\mathrm{T}} \;\; \cdots \;\; x_N^{\mathrm{T}} \,]^{\mathrm{T}}$, $A_i = [\, A_{i1} \;\; \cdots \;\; A_{ii-1} \;\; 0 \;\; A_{ii+1} \;\; \cdots \;\; A_{iN} \,]$; A_{ij}, B_i, D_i and C_i are constant matrices of appropriate dimensions; $\tau_i > 0$ is the time-delay, $\phi_i(t)$ is a continuous initial vector function, $\sum_{i=1}^N n_i = n$, $\sum_{i=1}^N r_i = r$ and $\sum_{i=1}^N m_i = m$. Assume that the reference input $\bar{y}_i$ to be tracked by y_i in the ith subsystem (1) is generated by an exosystem of the form

$$\begin{aligned} &\dot{z}_i(t) = F_i z_i(t) \\ &\bar{y}_i(t) = H_i z_i(t), \end{aligned} \tag{2}$$

where $z_i \in R^{p_i}$, $\bar{y}_i \in R^{m_i}$, F_i and H_i are constant matrices of appropriate dimensions, $\sum_{i=1}^N p_i = p$.

Remark 1. Many kinds of reference signals in control systems can be described by (2), e.g. step signals, ramp signals and periodic signals, etc. In most existing results, the exogenous signals are restricted to bounded ones. Since this paper considers an infinite-horizon optimal tracking control problem, we restrict the exosystem (2) to be asymptotically stable. The following assumptions are required such that the control problem is well posed.

Assumption 1. The pair (A_{ii}, B_i) is completely controllable and the pair (A_{ii}, C_i) is completely observable.

Assumption 2. All eigenvalues of F_i have negative real parts.

Assumption 3. The pair (F_i, H_i) is completely observable.

Assumption 4. H_i is a full-row-rank matrix.

The optimal tracking control problem is to design a controller to minimize the quadratic cost functional $J = \sum_{i=1}^{N} J_i$, where

$$J_i = \frac{1}{2}\int_0^{\infty}\left[e_i^{\mathrm{T}}(t)Q_i e_i(t) + u_i^{\mathrm{T}}(t)R_i u_i(t)\right]\mathrm{d}t, \tag{3}$$

where Q_i and R_i are positive-definite matrices of appropriate dimensions; $e_i(t)$ is the tracking error of each subsystem, i.e.

$$e_i(t) = \bar{y}_i(t) - y_i(t). \tag{4}$$

According to the necessary conditions of optimality, the optimal tracking control law of the ith subsystem (1) is given by

$$u_i^*(t) = -R_i^{-1}B_i^{\mathrm{T}}\lambda_i(t), \quad i = 1, 2, \dots N, \tag{5}$$

where $\lambda_i(t)$ is the solution to the following TPBV problem

$$\begin{aligned} &\dot{x}_i(t) = A_{ii}x_i(t) + A_i x(t) + D_i x_i(t-\tau_i) + B_i u_i(t), \quad t > 0 \\ &-\dot{\lambda}_i(t) = C_i^{\mathrm{T}}Q_i C_i x_i(t) - C_i^{\mathrm{T}}Q_i H_i z_i(t) + A_{ii}^{\mathrm{T}}\lambda_i(t) + D_i^{\mathrm{T}}\lambda_i(t+\tau_i) \\ &x_i(t) = \phi_i(t), \quad -\tau_i \le t \le 0 \\ &\lambda_i(\infty) = 0 \\ &i = 1, 2, \dots N. \end{aligned} \tag{6}$$

Note that it is coupled among N TPBV subproblems, in which both time-delay and time-advance terms are included. So it is extremely difficult to obtain the analytical solution in general.

3. Design of the optimal tracking controller

In order to prove the existence and uniqueness of the optimal tracking control law, a preliminary lemma is first introduced.

Lemma 1 ([14]). *The Sylvester equation with respect to $X \in R^{n\times m}$*

$$AX + XB = -C \tag{7}$$

has a unique solution if and only if

$$\lambda_\alpha(A) + \lambda_\beta(B) \ne 0, \quad \alpha = 1, 2, \dots, n, \ \beta = 1, 2, \dots, m, \tag{8}$$

where $A \in R^{n\times n}$, $B \in R^{m\times m}$ and $C \in R^{n\times m}$ are known matrices; $\lambda_i(\cdot)$ denotes the ith eigenvalue of the corresponding matrix.

Our main results are presented in the following theorem.

Theorem 1. *Consider the optimal tracking control problem (1)–(3) under Assumptions 1 and 2. The optimal tracking control law is existent and unique, and is given by*

$$\begin{aligned} &u^*(t) = \left[u_1^*(t)^{\mathrm{T}} \quad u_2^*(t)^{\mathrm{T}} \quad \cdots \quad u_N^*(t)^{\mathrm{T}}\right]^{\mathrm{T}} \\ &u_i^*(t) = -R_i^{-1}B_i^{\mathrm{T}}\left[P_i x_i(t) + L_i z_i(t) + \lim_{k\to\infty} g_i^{[k]}(t)\right], \quad i = 1, 2, \dots N, \end{aligned} \tag{9}$$

where P_i is the unique positive-definite solution to the following Riccati matrix equation

$$A_{ii}^{\mathrm{T}}P_i + P_i A_{ii} - P_i S_i P_i + C_i^{\mathrm{T}}Q_i C_i = 0, \tag{10}$$

L_i is the unique solution to the following Sylvester equation

$$(A_{ii} - S_i P_i)^{\mathrm{T}}L_i + L_i F_i = C_i^{\mathrm{T}}Q_i H_i, \tag{11}$$

$g_i^{[k]}(t)$ *is the solution to the following adjoint vector differential equation*

$$\begin{aligned} & g_i^{[0]}(t) \equiv 0 \\ & -\dot{g}_i^{[k]}(t) = (A_{ii} - S_i P_i)^{\mathrm{T}} g_i^{[k]}(t) + P_i \left[A_i x^{[k-1]}(t) + D_i x_i^{[k-1]}(t - \tau_i) \right] \\ & \qquad + D_i^{\mathrm{T}} \left[P_i x_i^{[k-1]}(t + \tau_i) + L_i z_i(t + \tau_i) + g_i^{[k-1]}(t + \tau_i) \right] \\ & g_i^{[k]}(\infty) = 0, \quad k = 1, 2, \ldots, \end{aligned} \tag{12}$$

where $x_i^{[k]}(t)$ *is the solution to the following state vector differential equation*

$$\begin{aligned} & x_i^{[0]}(t) \equiv 0 \\ & \dot{x}_i^{[k]}(t) = (A_{ii} - S_i P_i) x_i^{[k]}(t) - S_i L_i z_i(t) - S_i g_i^{[k]}(t) + D_i x_i^{[k-1]}(t - \tau_i) + A_i x^{[k-1]}(t), \quad t > 0 \\ & x_i^{[k]}(t) = \phi_i(t), \quad -\tau_i \le t \le 0, \ k = 1, 2, \ldots. \end{aligned} \tag{13}$$

Proof. Let

$$\lambda_i(t) = P_i x_i(t) + L_i z_i(t) + g_i(t), \quad i = 1, 2, \ldots N, \tag{14}$$

where $g_i : C^1(R_T) \to R^{n_i}$ is an adjoint vector to be solved, $R_T = (0, \infty)$. Taking the derivative of Eq. (14) along Eqs. (1) and (2) and substituting Eq. (5) into it gives

$$\begin{aligned} \dot{\lambda}_i(t) = {} & (P_i A_{ii} - P_i S_i P_i) x_i(t) + (L_i F_i - P_i S_i L_i) z_i(t) \\ & + P_i A_i x(t) + P_i D_i x_i(t - \tau_i) - P_i S_i g_i(t) + \dot{g}_i(t). \end{aligned} \tag{15}$$

Substituting Eq. (14) into the second equation of (6) yields

$$-\dot{\lambda}_i(t) = \left(A_{ii}^{\mathrm{T}} P_i + C_i^{\mathrm{T}} Q_i C_i \right) x_i(t) + \left(A_{ii}^{\mathrm{T}} L_i - C_i^{\mathrm{T}} Q_i H_i \right) z_i(t) + A_{ii}^{\mathrm{T}} g_i(t) + D_i^{\mathrm{T}} \lambda_i(t + \tau_i). \tag{16}$$

Adding the above two equations and comparing the coefficients give the Riccati matrix equation (10), the Sylvester equation (11) and the adjoint equation

$$\begin{aligned} & -\dot{g}_i(t) = (A_{ii} - S_i P_i)^{\mathrm{T}} g_i(t) + P_i A_i x(t) + P_i D_i x_i(t - \tau_i) + D_i^{\mathrm{T}} \lambda_i(t + \tau_i) \\ & g_i(\infty) = 0, \end{aligned} \tag{17}$$

where $S_i = B_i R_i^{-1} B_i^{\mathrm{T}}$. Under Assumption 1, there exists a unique positive-definite solution matrix P_i to Eq. (10). According to the optimal control theory, all eigenvalues of $A_{ii} - S_i P_i$ have negative real parts. Under Assumption 2, the following inequality holds

$$\lambda_\alpha (A_{ii} - S_i P_i) + \lambda_\beta(F_i) \neq 0, \quad \alpha = 1, 2 \ldots, n_i; \ \beta = 1, 2 \ldots, p_i. \tag{18}$$

By Lemma 1, there exists a unique solution matrix L_i to Eq. (11). Substituting Eq. (14) into Eq. (5) gives the following optimal tracking control law of the ith subsystem

$$u_i^*(t) = -R_i^{-1} B_i^{\mathrm{T}} \left[P_i x_i(t) + L_i z_i(t) + g_i(t) \right]. \tag{19}$$

Now the problem reduces to solving the adjoint vector g_i in Eq. (19). Substituting Eq. (19) into the first equation of (6) and Eq. (14) into Eq. (17), respectively, we can obtain

$$\begin{aligned} & \dot{x}_i(t) = (A_{ii} - S_i P_i) x_i(t) - S_i L_i z_i(t) - S_i g_i(t) + D_i x_i(t - \tau_i) + A_i x(t), \quad t > 0 \\ & -\dot{g}_i(t) = (A_{ii} - S_i P_i)^{\mathrm{T}} g_i(t) + P_i \left[A_i x(t) + D_i x_i(t - \tau_i) \right] \\ & \qquad + D_i^{\mathrm{T}} \left[P_i x_i(t + \tau_i) + L_i z_i(t + \tau_i) + g_i(t + \tau_i) \right] \\ & x_i(t) = \phi_i(t), \quad -\tau_i \le t \le 0 \\ & g_i(\infty) = 0. \end{aligned} \tag{20}$$

Note that it is coupled among N TPBV subproblems with both time-delay and time-advance terms in Eq. (20). So it is rather difficult to obtain the analytical or even the numerical solution. In order to obtain an approximate solution, we respectively construct a sequence of adjoint vector differential equations in (12), a sequence of state vector differential equations in (13), and the relevant control sequence

$$u_i^{[k]}(t) = -R_i^{-1} B_i^{\mathrm{T}} \left[P_i x_i^{[k]}(t) + L_i z_i(t) + g_i^{[k]}(t) \right]. \tag{21}$$

In the kth iteration, $x_i^{[k-1]}$, z_i and $g_i^{[k-1]}$ are known functions in Eq. (12). Therefore Eq. (12) is a nonhomogeneous linear vector differential equation and $g_i^{[k]}$ can be solved by reverse integration. With $g_i^{[k]}$ obtained, Eq. (13) is also a nonhomogeneous linear vector differential equation and $x_i^{[k]}$ can be easily solved.

By the results in [15,16], $\{x_i^{[k]}(t)\}$ and $\{g_i^{[k]}(t)\}$ uniformly converge to the solutions of the TPBV problem (20). $\{u_i^{[k]}(t)\}$ is only dependent on $\{x_i^{[k]}(t)\}$ and $\{g_i^{[k]}(t)\}$, so it is also uniformly convergent. As $k \to \infty$, the limit of $\{x_i^{[k]}(t)\}$ is the optimal state of the ith subsystem and the limit of $\{u_i^{[k]}(t)\}$ is the optimal tracking control law of the ith subsystem. This completes the proof. □

Remark 2. In practical design of the optimal tracking controller, the limit of $\{g_i^{[k]}(t)\}$ cannot be obtained precisely. It can be approximated by using the kth iteration of the adjoint vector sequence and the kth suboptimal tracking control law of the ith subsystem is given by

$$u_{ik}(t) = -R_i^{-1} B_i^{\mathrm{T}} \left[P_i x_i(t) + L_i z_i(t) + g_i^{[k]}(t) \right]. \tag{22}$$

Remark 3. Because $x_i(t)$ in the first term of Eq. (22) is the precise solution of the state vector, $u_{ik}(t)$ is better than the kth approximate optimal tracking control law given by Eq. (21). In practice, k can be chosen according to the control precision requirement.

Next, an algorithm is given to calculate the kth suboptimal tracking control law.

Algorithm 1. *Step* 1: Obtain $\bar{y}_i(t)$ from Eq. (2). Solve Eqs. (10) and (11) for P_i and L_i, respectively. Let $k = 1$. Give a constant ε and a sufficiently large positive number $J^{[0]}$.
Step 2: Solve Eq. (12) for $g_i^{[k]}(t)$. Substitute $g_i^{[k]}(t)$ into Eq. (22) and calculate $u_{ik}(t)$.
Step 3: Substitute $u_{ik}(t)$ into the ith subsystem (1) and obtain the closed-loop system. Obtain $e_i^{[k]}(t)$ from Eq. (4) and calculate $J^{[k]}$ from the following formula

$$J^{[k]} = \sum_{i=1}^{N} J_i^{[k]}, \tag{23}$$

where

$$J_i^{[k]} = \frac{1}{2} \int_0^{\infty} \left[e_i^{[k]}(t)^{\mathrm{T}} Q_i e_i^{[k]}(t) + u_{ik}(t)^{\mathrm{T}} R_i u_{ik}(t) \right] \mathrm{d}t. \tag{24}$$

Step 4: If $|(J^{[k-1]} - J^{[k]})/J^{[k]}| < \varepsilon$, then output $u_{ik}(t)$ and stop calculation.
Step 5: Substitute $g_i^{[k]}(t)$ into Eq. (13) and solve $x_i^{[k]}(t)$.
Step 6: Let $k = k + 1$ and go to Step 2.

Remark 4. The proposed algorithm only requires solving the Riccati equation (10) and the Sylvester equation (11) once, and mainly solves an iteration formula of adjoint vector sequence. It takes less computing time and memory space compared with iteration of matrix differential equations. Therefore, it has a computational advantage and is more promising for on-line computation.

4. Design of the reference input observer

Note that the optimal tracking control law (9) contains the state variable $z_i(t)$ of the exosystem, which is physically unrealizable. In this section, a reduced-order reference input observer is designed to make the feedforward term physically realizable.

Under Assumption 4, for the full-row-rank matrix H_i, there exists a constant matrix $M_i \in R^{(p_i-m_i)\times p_i}$ such that $\begin{bmatrix} H_i^{\mathrm{T}} & M_i^{\mathrm{T}} \end{bmatrix} \in R^{p_i\times p_i}$ is nonsingular. Let

$$T_i = \begin{bmatrix} H_i \\ M_i \end{bmatrix}^{-1} = \begin{bmatrix} T_{i1} & T_{i2} \end{bmatrix}, \qquad T_i^{-1} F_i T_i = \begin{bmatrix} F_{i1} & F_{i12} \\ F_{i21} & F_{i2} \end{bmatrix} \tag{25}$$

where $T_{i1} \in R^{p_i\times m_i}$, $T_{i2} \in R^{p_i\times(p_i-m_i)}$, $F_{i1} \in R^{m_i\times m_i}$, $F_{i12} \in R^{m_i\times(p_i-m_i)}$, $F_{i21} \in R^{(p_i-m_i)\times m_i}$ and $F_{i2} \in R^{(p_i-m_i)\times(p_i-m_i)}$. In order to construct a reference input observer, we make a nonsingular transformation $z_i = T_i\bar{z}_i$. Denote $\bar{z}_i^{\mathrm{T}} = \begin{bmatrix} \bar{z}_{i1}^{\mathrm{T}} & \bar{z}_{i2}^{\mathrm{T}} \end{bmatrix}$, where $\bar{z}_{i1} \in R^{m_i}$ and $\bar{z}_{i2} \in R^{(p_i-m_i)}$. An equivalent system to the exosystem (2) is obtained as follows:

$$\begin{aligned} \dot{\bar{z}}_{i1}(t) &= F_{i1}\bar{z}_{i1}(t) + F_{i12}\bar{z}_{i2}(t) \\ \dot{\bar{z}}_{i2}(t) &= F_{i21}\bar{z}_{i1}(t) + F_{i2}\bar{z}_{i2}(t) \\ \bar{y}_i(t) &= \bar{z}_{i1}(t). \end{aligned} \tag{26}$$

In Eq. (26), note that $\bar{z}_{i1}$ is just the reference input $\bar{y}_i$. So we only need to construct a reduced-order observer with respect to $\bar{z}_{i2}$. Under Assumption 3 and the fact that $H_iT_i = \begin{bmatrix} I_{im} & 0 \end{bmatrix}$, it can be proved that the pair (F_{i2}, F_{i12}) is also completely observable. The reference input observer is designed as follows:

$$\begin{aligned} \dot{w}_i(t) &= \hat{F}_i w_i(t) + \hat{H}_i\bar{y}_i(t) \\ \hat{z}_{i2}(t) &= w_i(t) + G_i\bar{y}_i(t), \end{aligned} \tag{27}$$

where $w_i \in R^{(p_i-m_i)}$ is the constructed state vector of the observer; $\hat{F}_i = F_{i2} - G_iF_{i12}$, $\hat{H}_i = F_{i2}G_i - G_iF_{i12}G_i + F_{i21} - G_iF_{i1}$; $\hat{z}_{i2}$ is the observing value of $\bar{z}_{i2}$; G_i is a coefficient matrix to be determined. In order to guarantee the convergence rate and precision of the observer, G_i can be chosen to assign all eigenvalues of $F_{i2} - G_iF_{i12}$ at desired locations. Combining Eq. (2) with Eqs. (25)–(27) gives the observing value of $z_i(t)$

$$\hat{z}_i(t) = T_{i2}w_i(t) + (T_{i1} + T_{i2}G_i)\bar{y}_i(t). \tag{28}$$

By the above reconstruction of $z_i(t)$, a dynamical suboptimal tracking control law of the ith subsystem is given as

$$\begin{aligned} \dot{w}_i(t) &= \hat{F}_i w_i(t) + \hat{H}_i\bar{y}_i(t) \\ u_{ik}(t) &= -R_i^{-1}B_i^{\mathrm{T}}\left[P_ix_i(t) + L_iT_{i2}w_i(t) + L_i(T_{i1} + T_{i2}G_i)\bar{y}_i(t) + g_i^{[k]}(t)\right]. \end{aligned} \tag{29}$$

5. A numerical example

Consider the optimal tracking control problem described by Eqs. (1)–(3), where

$$\begin{aligned} &A_{11} = \begin{bmatrix} -0.5 & 3 \\ -0.5 & 2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.8 & 1 \\ -3 & 2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} -1 & 0 \\ 0.8 & -1.5 \end{bmatrix} \\ &A_{22} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -0.8 & 0 \\ 0.6 & -1 \end{bmatrix} \\ &B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 3 & 0 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0 & 5 \\ -0.8 & -2 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \\ &B_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 2 & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 2 \\ -0.5 & -1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 2 & 0 \end{bmatrix} \\ &\phi_1(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}, \quad z_1(0) = \begin{bmatrix} 1 & 0.8 \end{bmatrix}^{\mathrm{T}}, \quad Q_1 = 2, \quad R_1 = 1 \\ &\phi_2(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}, \quad z_2(0) = \begin{bmatrix} 0.4 & 0.4 \end{bmatrix}^{\mathrm{T}}, \quad Q_2 = 2, \quad R_2 = 1. \end{aligned} \tag{30}$$

The control precision is set at $\varepsilon = 0.05$. When $|(J^{[k-1]} - J^{[k]})/J^{[k]}| < \varepsilon$, $u_{ik}(t)$ is viewed as the suboptimal tracking control law of the ith subsystem.

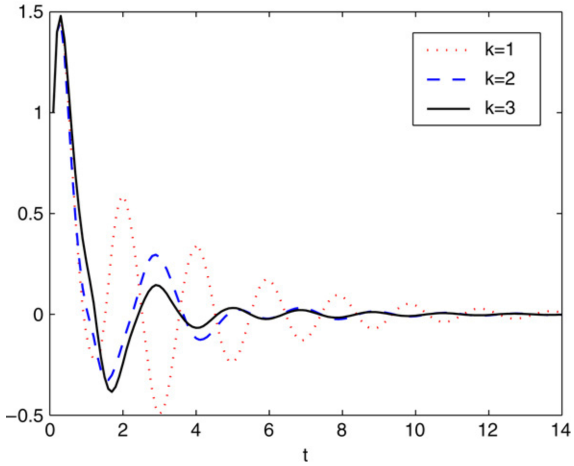

Fig. 1. Simulation curves of the tracking error $e_1^{[k]}(t)$ when $k = 1, 2, 3$.

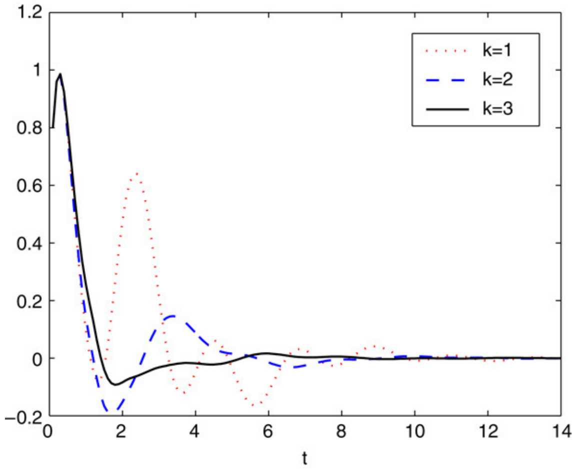

Fig. 2. Simulation curves of the tracking error $e_2^{[k]}(t)$ when $k = 1, 2, 3$.

Table 1
The cost functional values and relative errors

k	1	2	3	4
$J^{[k]}$	4.8240	2.9758	2.7417	2.7153
$\lvert(J^{[k-1]} - J^{[k]})/J^{[k]}\rvert$	–	0.6211	0.0854	0.0097

In Figs. 1–4 are presented the simulation curves of the tracking errors and control inputs when the time-delays $\tau_1 = \tau_2 = 1$. Listed in Table 1 are the cost functional values and relative errors at each iteration step. From Figs. 1–4, it can be seen that the more iterations we take, the better the approximations are. Table 1 shows that $J^{[1]} > J^{[2]} > J^{[3]} > J^{[4]}$, which means that the cost functional value decreases as the iteration step increases, and approaches a stable optimal value J^*. Table 1 also shows that the relative error of the cost functional value decreases with an increase in iteration step. When $k = 4$, it satisfies the control precision requirement and $u = \begin{bmatrix} u_{14}^{\mathrm{T}}(t) & u_{24}^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$ can be viewed as the suboptimal tracking control law.

The above numerical example shows that the presented algorithm has a relatively fast convergence rate and it only requires a few iterations to yield the suboptimal tracking control law.

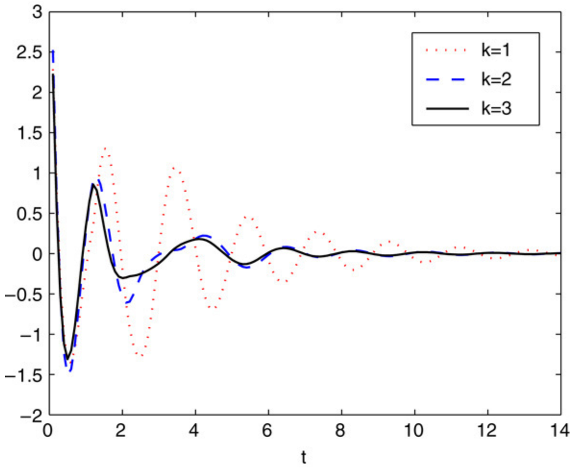

Fig. 3. Simulation curves of the control input $u_1^{[k]}(t)$ when $k = 1, 2, 3$.

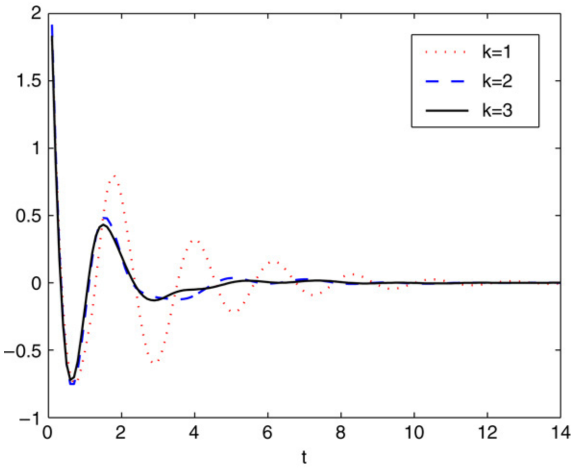

Fig. 4. Simulation curves of the control input $u_2^{[k]}(t)$ when $k = 1, 2, 3$.

6. Conclusion

This paper has proposed an infinite-horizon optimal tracking controller for a class of large-scale interconnected systems with state time-delays. The successive approximation approach has been applied to avoid solving the TPBV problem with both time-delay and time-advance terms directly. The presented algorithm requires less computing time and memory space and is more promising for on-line computation. The numerical example illustrates that this approach is effective and easy to implement.

Acknowledgements

The authors would like to thank the editors and the anonymous reviewers for their constructive comments and suggestions to improve the quality of the paper.

This work was partially supported by the National Natural Science Foundation of China (Grant No. 60574023), the Natural Science Foundation of Shandong Province (Grant No. Z2005G01), and the Natural Science Foundation of Qingdao City (Grant No. 05-1-JC-94).

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